Topological Defects and Inflation

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Abstract

In the context of supersymmetric models, we analyze the production of topological defects at the end of inflation driven by a conjugate pair of inflaton fields which are non-singlets under the continuous symmetry group of the theory. We find that magnetic monopoles of mass on the order of 10^{13} GeV can survive inflation and be present in our galaxy at an observable level. We also consider cosmic strings as well as domain walls.

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The new and the chaotic inflationary scenarios [1] typically invoke a very weakly coupled scalar field known as the inflaton. The extremely weak couplings of this field are necessitated by the small value of the observed temperature fluctuations in the cosmic background radiation. Within the context of ordinary grand unified theories (GUTS), the inflaton is required to be a gauge singlet field. This avoids the 'strong' radiative corrections that the gauge interactions would otherwise produce. Supersymmetric (SUSY) GUTS provide us with the possibility of using as inflaton even a gauge non-singlet weakly coupled field [2]. The idea is to utilize the very same conjugate pair of standard model (SM) singlet superfields which is also responsible for the breaking of the gauge symmetry of the theory. Mutual cancellation of the D-terms of these conjugate fields can then be easily achieved along some direction and guarantees the absence of strong radiative corrections which can spoil the smallness of higgs self-couplings in this D-flat direction.

The intriguing idea that the inflaton is a gauge non-singlet field raises the interesting possibility that topological defects like magnetic monopoles or cosmic strings can be produced at the end of inflation. In ref. 2, we presented preliminary estimates of topological defect production by a gauge non-singlet inflaton in the context of a particular class of superstring theories. The purpose of this paper is to give, in the context of SUSY GUTS, a detailed analysis of topological defect production at the end of inflation driven by a conjugate pair of superfields which are non-singlets under the continuous (gauge and global) symmetry group of the theory.

We consider a SUSY GUT based on a gauge group G. The theory possibly possesses some global symmetries too which may include both continuous

and discrete parts. Symmetry breaking is obtained through a superpotential which includes the terms

$$W = \kappa S(-M^2 \ + \ \phi\bar{\phi}). \tag{1}$$

Here $\phi, \bar{\phi}$ is a conjugate pair of left-handed SM singlet superfields which belong to non-trivial representations of the continuous (gauge and global) symmetry group of the theory and reduce its rank by their vacuum expectation values (vevs), S is a gauge singlet left-handed superfield, M is a superheavy mass scale and κ a positive coupling constant. The superpotential terms in eq. (1) are the only renormalizable couplings which involve the superfields $S, \phi, \bar{\phi}$ and are consistent with a continuous R-symmetry under which $W \to e^{i\theta}W,\ S \to e^{i\theta}S$ and $\phi\bar{\phi} \to \phi\bar{\phi}$. The non-renormalizable terms $S(\phi\bar{\phi})^n$ $(n \geq 2)$ which are also allowed by this symmetry are assumed to be small and will be ignored in our discussion. Moreover, we assume that the presence of other SM singlets in the theory does not affect the superpotential in eq. (1) which is responsible for the non-zero vevs of $\phi, \bar{\phi}$. The potential obtained from W in eq. (1) is

$$V = \kappa^2 \mid M^2 - \phi\bar{\phi} \mid^2 \ + \kappa^2 \mid S \mid^2 (\mid \phi \mid^2 + \mid \bar{\phi} \mid^2) \ + \ D - terms, \tag{2}$$

where the scalar components of the superfields are denoted by the same symbols as the corresponding superfields. Vanishing of the D-terms is achieved with $\mid \bar{\phi} \mid = \mid \phi \mid$ (D-flatness condition). The supersymmetric vacuum

$$< S >= 0,\ < \phi >< \bar{\phi} >= M^2,\ \mid < \bar{\phi} > \mid = \mid < \phi > \mid \tag{3}$$

lies on the D-flat direction $\bar{\phi}^* = \phi$. Restricting ourselves to this particular direction and performing appropriate continuous R- and non-R-transformations, we can bring the complex $S, \phi, \bar{\phi}$ fields on the real axis, i.e., $S \equiv \frac{\sigma}{\sqrt{2}}$, $\bar{\phi} = \phi \equiv \frac{1}{2}\chi$, where σ and χ are normalized real scalar fields. The potential in eq. (2) then takes the form

$$V(\chi, \sigma) \; = \; \kappa^2 \left(M^2 - \frac{\chi^2}{4} \right)^2 \; + \; \frac{\kappa^2 \sigma^2 \chi^2}{4} \tag{4}$$

and the supersymmetric minima become

$$< \chi >= \pm 2M \quad , \quad < \sigma >= 0. \tag{5}$$

It should be noted that inflation and topological defect production with potentials of the type in eq. (4) were first studied in ref. 3. However, these studies were restricted to non-supersymmetric models and used initial field configurations different than the ones considered here.

The vacuum manifold of the theory, which is obtained from the minimum in eq. (5) by performing all possible gauge and global transformations, may have non-trivial homotopical properties in which case the theory predicts the existence of topological defects.

Following the philosophy of the chaotic inflationary scenario [1], we suppose that at a cosmic time $t_P \equiv M_P^{-1}$, where $M_P = 1.2 \times 10^{19}$ GeV is the Planck mass, the universe emerges with energy density of order M_P^4 and we concentrate on a particular region of size of order t_P where χ and σ happen to be almost uniform with $\mid \chi \mid \gg \mid \sigma \mid$. The potential in eq. (4) is then initially dominated by the χ^4 term and the initial equations of motion for

the χ and σ fields read

$$\ddot{\chi} \ + \ 3H\dot{\chi} \ + \ \frac{\kappa^2\chi^3}{4} \ \simeq \ 0 \tag{6}$$

and

$$\ddot{\sigma} \ + \ 3H\dot{\sigma} \ + \ \frac{\kappa^2\chi^2\sigma}{2} \ \simeq \ 0, \tag{7}$$

where

$$H \ = \ \left(\frac{8\pi}{3}\right)^{1/2} M_P^{-1} \rho^{1/2} \ = \ \left(\frac{8\pi}{3}\right)^{1/2} M_P^{-1} \left(\frac{1}{2}\dot{\chi}^2 \ + \ \frac{1}{2}\dot{\sigma}^2 \ + \ V(\chi,\sigma)\right)^{1/2} \tag{8}$$

is the Hubble parameter, the dots denote derivatives with respect to cosmic time and ρ is the energy density. It is easily seen that for $\chi \gg M_P/(3\pi)^{1/2}$, eqs. (6) and (7) reduce to

$$3H\dot{\chi} \ + \ \frac{\kappa^2\chi^3}{4} \ \simeq \ 0 \tag{9}$$

and

$$3H\dot{\sigma} \ + \ \frac{\kappa^2\chi^2\sigma}{2} \ \simeq \ 0 \tag{10}$$

respectively, while the kinetic terms of the χ and σ fields in the RHS of eq. (8) can be neglected. During this inflationary period, eqs. (9) and (10) give

$$\chi \ = \ \chi_o \ exp\left(-\frac{\kappa M_P \Delta t}{(24\pi)^{1/2}}\right) \tag{11}$$

and

$$\sigma \ = \ \sigma_o \ exp\left(-\frac{2\kappa M_P \Delta t}{(24\pi)^{1/2}}\right), \tag{12}$$

where χ_o and σ_o are the initial values of the fields at time $t_P(\chi_o \gg \sigma_o)$ and $\Delta t \equiv t - t_P$. Eqs. (11) and (12) imply that $\chi^2/\sigma = \chi_o^2/\sigma_o$ and the ratio σ/χ decreases exponentially with cosmic time. From eq. (9), the number of e-foldings, $N(\chi)$, from when the field has value χ till inflation ends turns out to be

$$N(\chi) \ = \ \pi \, \frac{\chi^2}{M_P^2} \ . \tag{13}$$

The contribution of the scalar metric perturbation to the microwave background quadrupole anisotropy (scalar Sachs-Wolfe effect) is given by [4]

$$\left(\frac{\Delta T}{T}\right)_S \ \simeq \ \left(\frac{32\pi}{45}\right)^{1/2} \ \frac{V^{3/2}}{M_P^3 \, (\partial V/\partial \chi)} \Bigg|_{k\sim H} \ = \left(\frac{32\pi}{45}\right)^{1/2} \ \frac{\kappa \chi^3}{16 M_P^3}\Bigg|_{k\sim H} , \tag{14}$$

where the right-hand side is evaluated at the value of the χ field where the length scale k^{-1}, which corresponds to the present horizon size, crossed outside the de Sitter horizon during inflation. Substituting χ from eq. (13) in eq. (14), we obtain

$$\left(\frac{\Delta T}{T}\right)_S \ \simeq \ \left(\frac{32}{45}\right)^{1/2} \ \frac{\kappa}{16\pi} \ N_H^{3/2}, \tag{15}$$

where N_H is the number of e-foldings of the present horizon size during inflation. The gravitational wave contribution $\left(\frac{\Delta T}{T}\right)_T$ to the quadrupole anisotropy is [5]

$$\left(\frac{\Delta T}{T}\right)_T \ \simeq \ 0.78 \frac{V^{1/2}}{M_P^2} \ = \ 0.78 \frac{\kappa}{4\pi} N_H. \tag{16}$$

Taking $N_H = 55$ and $(\Delta T/T) \simeq 7 \times 10^{-6}$ from COBE, we then obtain $\kappa \simeq 0.92 \times 10^{-6}$ and $r \equiv \left(\frac{\Delta T}{T}\right)_T^2 / \left(\frac{\Delta T}{T}\right)_S^2 \simeq 0.25$. The spectral index

$$n = 1 - \frac{3}{N_H} \simeq 0.945 \tag{17}$$

turns out to be very close to the Harrison-Zeldovich value ($n = 1$) and lies in the central range of values preferred by observations.

At the end of inflation at cosmic time $t_f \sim H_f^{-1} \sim 3(6\pi)^{1/2}(\kappa M_P)^{-1}$, the χ field starts performing damped oscillations over its maximum at $\chi = 0$ with frequency of order $\kappa\chi_m$, where χ_m is the amplitude of these oscillations. As is well known [6], an oscillating field with χ^4 potential behaves like radiation, i.e., $\rho + p = (4/3)\rho$, where p is the pressure averaged over one oscillation time of this field. From eq. (8) and the continuity equation $\dot{\rho} = -3H(\rho + p)$, we obtain $H \simeq 1/2t$ and $\chi_m \simeq (3/2\pi)^{1/4} \, (M_P/\kappa t)^{1/2}$. After inflation is over, eq. (7) averaged over one oscillation of χ takes the form

$$\ddot{\sigma} + \frac{3}{2t}\dot{\sigma} + \left(\frac{3}{32\pi}\right)^{1/2} \frac{\kappa M_P}{t}\sigma \simeq 0. \tag{18}$$

For $t \gg t_f$, the 'frequency' of the σ field is of order

$$\left(\frac{3}{32\pi}\right)^{1/4} \left(\frac{\kappa M_P}{t}\right)^{1/2} \tag{19}$$

and is much greater than $H \simeq 1/2t$. Thus, σ also starts performing damped oscillations about $\sigma = 0$ but with an initial amplitude much smaller than the amplitude of χ. The presence of the σ field is not essential for our subsequent arguments and, for simplicity, we will put it equal to zero in the rest of the discussion.

Topological defects associated with the symmetry breaking caused by the non-zero vev of the χ field assumed to be a gauge non-singlet can, in principle, be produced at cosmic time $t_d \sim H_d^{-1} \ = \ (3/8\pi)^{1/2} \ (M_P/\kappa M^2)$, where the energy density ρ of the oscillating χ field reduces to the value $\kappa^2 M^4$. At this point the χ field ceases to oscillate over the potential barrier $V(\chi = 0, \ \sigma = 0) \ = \ \kappa^2 M^4$ at $\chi = 0$ and gets trapped in one or the other of the potential wells associated with the two minima in eq. (5). As we explained earlier, the oscillations of the χ field, for cosmic times between t_f and t_d, are approximately governed by a χ^4 potential and the system behaves almost like radiation. This means that any density fluctuations at scales smaller than the horizon at t_d are erased. So, at cosmic time t_d, the χ field is expected to be uniform on scales smaller than about t_d and therefore fall into the same potential well everywhere within a particle horizon. Thus, the smallest possible distance between neighbouring defects produced at t_d is of order t_d. This maximal number of topological defects per horizon at t_d is achieved [7] only if the inflationary density fluctuation on scale t_d, $(\delta\rho/\rho)_d$, exceeds the fraction of energy density lost, $(\delta\rho_{1/2}/\rho)$, within half a cycle of oscillation of the χ field at t_d. This energy loss can be calculated by using the equation of continuity $\dot{\rho} = -3H\dot{\chi}^2$ and eqs. (8) and (4),

$$\begin{aligned} \frac{\delta\rho_{1/2}}{\rho} &= \frac{6H}{\kappa^2 M^4} \int_o^{2\sqrt{2}M} \dot{\chi} d\chi = \frac{6H}{\kappa^2 M^4} \int_o^{2\sqrt{2}M} \sqrt{2(\rho - V)} \, d\chi \\ &= 16 \left(\frac{8\pi}{3}\right)^{1/2} \frac{M}{M_P} \ . \end{aligned} \tag{20}$$

Moreover, the inflationary density fluctuation is

$$\left(\frac{\delta\rho}{\rho}\right)_d \simeq \left(\frac{\delta\rho}{\rho}\right)_H \left(\frac{N_d}{N_H}\right)^{3/2}, \tag{21}$$

where [1]

$$\left(\frac{\delta\rho}{\rho}\right)_H \simeq 9.3\left(\frac{\Delta T}{T}\right)_S \simeq 5.8 \times 10^{-5} \tag{22}$$

is the density perturbation on the present horizon scale and N_d is the number of e-foldings which the particle horizon at t_d suffered during inflation. To estimate N_d, recall that between t_f and t_d our system behaves like radiation and the scale factor of the universe increases by a factor $(V(\chi = \chi_f)/\kappa^2 M^4)^{1/4} = \chi_f/2M = (12\pi)^{-1/2}(M_P/M)$. The horizon size at t_d at the end of inflation is $(3/\sqrt{2})(\kappa M)^{-1}$. Comparing this with H_f^{-1} we find $exp(N_d) \simeq (12\pi)^{-1/2}(M_P/M)$. Eqs. (20), (21), (22) and the condition for 'maximal' production of topological defects at t_d then imply that $M \leq 1.9 \times 10^{12}$ GeV or $< \chi > \leq 3.8 \times 10^{12}$ GeV.

Let us first consider the case of magnetic monopoles. Their initial number density, if they are 'maximally' produced at t_d, is expected to be $n_M \sim t_d^{-3}$ and so the relative initial energy density in monopoles is

$$d_M = \frac{n_M m_M}{\kappa^2 M^4} \sim \left(\frac{8\pi}{3}\right)^{3/2} \frac{\kappa M^2 m_M}{M_P^3}, \tag{23}$$

where m_M is the monopole mass. At cosmic times greater than t_d, the χ field performs damped coherent oscillations about the minima in eq. (5). The potential is now approximately quadratic, the system behaves like matter and so d_M remains constant. The oscillating inflaton will eventually decay into lighter particles and 'reheat' the universe to temperature T_r. The process

of 'reheating' is strongly dependent on the particle physics model one adopts, but bounds can be obtained in a relatively model independent way. Assume that the inflaton field, which has mass $m_\chi = \sqrt{2}\kappa M$, decays to a pair of particles (say fermions) with mass $m \leq m_\chi/2$. The relevant effective coupling constant $f \leq m/<\chi>$. The decay rate $\Gamma \sim f^2 m_\chi \leq (1/4\sqrt{2})\kappa^3 M$ and the 'reheat' temperature $T_r \sim (\Gamma M_P)^{1/2} \leq 2^{-5/4}\kappa^{3/2}(M M_P)^{1/2}$. At 'reheat', $d_M \simeq n_M m_M / s T_r$, where s is the entropy density and, from eq. (23),

$$\frac{n_M}{s} \simeq \left(\frac{8\pi}{3}\right)^{3/2} \frac{\kappa M^2 T_r}{M_P^3} \leq 2^{-5/4}\left(\frac{8\pi}{3}\right)^{3/2}\left(\frac{\kappa M}{M_P}\right)^{5/2} \leq 10^{-31} \,. \quad (24)$$

Thus, a flux of intermediate mass ($\sim 10^{13}$ GeV) magnetic monopoles close to the Parker bound may exist in our galaxy.

Turning now to cosmic strings, the string network, which enters the particle horizon at some cosmic time subsequently approaches the well-known scaling solution [8]. The requirement that a string network exists in the present universe means that the scale of this network at production time is greater than or equal to t_d, and must not exceed the scale of the present universe at t_d. This is achieved if $(\delta\rho/\rho)_H \gtrsim \delta\rho_{1/2}/\rho$, which gives $M \leq 1.5 \times 10^{13}$ GeV or $<\chi> \leq 3 \times 10^{13}$ GeV. These strings presumably are too 'light' to play any role in galaxy formation.

Domain walls are cosmologically catastrophic and must be avoided. Their case is very similar to the cosmic string case we have just discussed and one finds that the problem is avoided if $<\chi> \geq 3 \times 10^{13}$ GeV. So, if the non-zero vev of the inflaton χ breaks some discrete symmetries, we should make

sure that this vev is of 'superheavy' ($\gg 10^{12}$ GeV) scale.

Our analysis can be extended to the inflationary scenario of ref. 2 where the inflaton ϕ, being a gauge non-singlet, is responsible for the 'radiative' breaking of the $SU(3)^3$ gauge symmetry of the theory. The relevant potential consists of a negative mass^2 term characterized by the supersymmetry breaking scale $M_s \sim$ TeV, and a non-renormalizable term (ϕ^6/M_P^2) with a small dimensionless coupling fixed by $\delta\rho/\rho$. The the vev of ϕ is determined to be on the order of the GUT scale so that topological defects do not arise.

In summary, in the framework of supersymmetric models, we estimated the density of topological defects that can be produced at the end of an inflationary epoch driven by a conjugate pair of gauge non singlet fields. We find that magnetic monopoles close to the Parker bound and/or cosmic strings may be produced if the inflaton vev takes intermediate scale values.

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